

Complete the Square

Compute the integral $\int \frac{dx}{\sqrt{2x - x^2}}$ by completing the square.

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$$\int \frac{dx}{\sqrt{2x-x^2}}$$

$$= \int \frac{dx}{\sqrt{-(x-1)^2+1}}$$

$$= \int \frac{du}{\sqrt{-u^2+1}}$$

$$= \int \frac{-\sin \theta}{\sqrt{-\cos^2 \theta + 1}} d\theta$$

$$= - \int \frac{\sin \theta}{\sqrt{\sin^2 \theta}} d\theta$$

$$= - \int d\theta$$

$$= -\theta + C$$

$$= -\arccos u + C$$

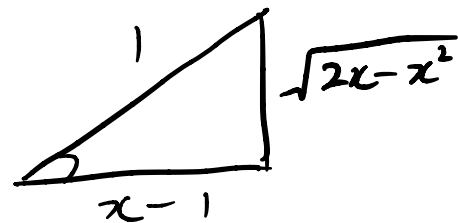
$$= -\arccos(x-1) + C$$

$$\begin{aligned} 2x-x^2 &= -(x^2-2x+(1)^2-(1)^2) \\ &= -(x-1)^2+1 \end{aligned}$$

Let $u = x-1$.

and $u = \cos \theta$.

$$\Rightarrow du = -\sin \theta d\theta$$



$$y = \arccos(x-1)$$

$$\cos y = x-1$$

$$-\sin y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{2x-x^2}}$$